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A DYNAMIC SUBGRID-SCALE MODEL FOR LOW-REYNOLDS-NUMBER CHANNEL FLOW

HONG ZHAO

Department of Transport Technology, Loughborough University of Technology, Loughborough, U.K.

AND

PETER R. VOKE

Department of Mechanical Engineering, University of Surrey, Guildford, U.K.

SUMMARY

Several issues related to applications of the dynamic subgrid-scale (SGS) model in large-eddy simulation (LES) at low Reynolds number are investigated. A modified formulation of the dynamic model is constructed and its performance in low-Reynolds-number LES of channel flow is assessed through a comparison of length scales computed respectively by this modified model, the Germano–Lilly dynamic SGS model and two empirical wall damping functions with optimum model coefficients, which have been successfully used in many simulations of channel flows. Two values of the ratio of filter widths are set for each of the dynamic models. The results have confirmed that the modified dynamic SGS model gives the correct behaviour of the subgrid eddy viscosity in the region of a plane wall to an accuracy that exceeds the best-tuned wall damping function, and almost collapses with the theoretical behaviour of the length scale near the wall without any tuning and adjustment. In addition, the impact of the choice of the ratio of filter widths on the modified dynamic SGS model is found to be much less than with the Germano–Lilly model.

KEY WORDS turbulence; channel flow; low Reynolds number; large-eddy simulation; subgrid-scale model; dynamic SGS model

1. INTRODUCTION

With the great advances in the availability of modern supercomputers, interest in performing large-eddy simulations of engineering flows is increasing. In such simulations, SGS modelling will play an even more important role and conventional, fixed coefficient SGS models such as the Smagorinsky model are unlikely to be satisfactory.

Recently a new class of SGS model called the dynamic model was developed by Germano *et al.*[1] and modified by Lilly[2] (hereafter referred to as the Germano–Lilly model). Following Germano's pioneering work,[1] many researchers have applied this model to complex flows where the Smagorinsky model does not work successfully, e.g. backward-facing step flows,[3] turbulent cavity flows,[4] stratified Ekman layers,[5] turbulent recirculating flows[6] and rotating turbulent flows.[7] Some authors also extended the model to simulate compressible turbulence,[8,9] though the model was derived from incompressible turbulence, and their results showed that the dynamic model is able to provide good descriptions of highly compressible turbulence. All the results have illustrated generally better agreement with direct numerical simulation (DNS) databases and experimental results.

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The success of the dynamic SGS model is mainly due to its most promising feature that the model coefficient is computed by using the information of the smallest resolved scales, in principle at each point in space and at each instant in the time integration. This leads to two superior features. Firstly, the model coefficient is computed dynamically as a part of the solution and therefore is adjustable by itself according to flow conditions from flow to flow. Secondly, the model coefficient is possibly negative at some locations in the flow domain or at some time instant during the time integration and thus is capable of accounting for energy backscatter from the subgrid scales to the resolved scales. The Germano–Lilly model is able to provide a way of circumventing most of the deficiencies related to the Smagorinsky model.

(a) The Smagorinsky model coefficient is input *a priori*. This single universal constant is incapable of representing correctly various turbulent flows.
(b) An *ad hoc* damping function must be used to obtain modelled SGS stresses with proper asymptotic near-wall behaviour.
(c) The length scale to be used with an anisotropic grid is unclear.
(d) The eddy viscosity does not vanish in the laminar regime.
(e) The backscatter of energy transfer is ruled out completely.

However, the success of the dynamic model in low-Reynolds-number LES is marginal. Cabot and Moin[10] and the present study show that low-Reynolds-number LES results for channel flow using the Germano–Lilly dynamic model are more sensitive to the choice of the ratio of the test filter width to the grid filter width, despite Germano's finding in his test case that the results are insensitive to the value of that ratio. In addition, it was found from the results of low-Reynolds-number LES for channel flow that the Germano–Lilly model encounters more numerical instabilities and the resulting model coefficient has a higher magnitude in the central region of the channel.

The main aim of this paper is to construct a modified dynamic model and to evaluate its performance by comparing its results with those from the Germano–Lilly model and with theoretical results. Our model provides a straightforward and physically meaningful averaging method to solve the problem of numerical instabilities and alleviates the deficiency of high magnitude of the computed model coefficient in the central region of the channel. A number of further issues regarding the application of the dynamic model to LES are discussed, such as the appropriate form of averaging method to remove the numerical instability, the choice of the test filter width and the ability to account for energy backscatter.

2. SUBGRID-SCALE MODELLING

2.1. *Germano–Lilly dynamic subgrid-scale model*

Germano *et al.*[1] derived the formulation of the dynamic SGS model by making judicious use of a nested grid, which is obtained through filtering the velocity field twice, first by a grid filter and then by a coarser filter called the test filter. For an incompressible flow, the dynamically computed model coefficient C involved in the computation of the eddy viscosity $\nu_t = C\Delta^2\overline{S}$, which is equivalent to C_s^2, the square of the original Smagorinsky constant, is given as[1,11]

$$C = \frac{L_{kl}\overline{S}_{kl}}{2\hat{\Delta}^2\hat{\overline{S}}\hat{\overline{S}}_{ij}\overline{S}_{ij} - 2\Delta^2\widehat{\overline{S}\overline{S}_{ij}}\overline{S}_{ij}}, \tag{1}$$

with $\bar{S}_{ij} = \frac{1}{2}(\partial\bar{u}_i/\partial x_j + \partial\bar{u}_j/\partial x_i)$ and $\bar{S}^2 = 2\bar{S}_{ij}\bar{S}_{ij}$, where $L_{ij} = \widehat{\bar{u}_i\bar{u}_j} - \hat{\bar{u}}_i\hat{\bar{u}}_j$ are called resolved stresses, since they are resolvable from the filtered velocities, and Δ and $\hat{\Delta}$ are the grid and test filter widths respectively.

Later Lilly[2] modified the dynamic SGS model to overcome two drawbacks. Firstly, the selection of $\bar{S}_{ij}$ for the contracting tensor to obtain a scalar equation (1) for C is arbitrary. Secondly, numerical instabilities are encountered, because the denominator of the formulation of C can vanish or become very small. By employing a least squares technique, he obtained a natural choice for the contracting tensor needed to derive a scalar equation for solving C. Thus the model coefficient C (the Germano–Lilly model coefficient) can be unambiguously written as

$$C = \frac{L_{kl}M_{kl}}{2M_{ij}M_{ij}}, \tag{2}$$

where

$$M_{ij} = \left(\hat{\Delta}^2\hat{\bar{S}}\hat{\bar{S}}_{ij} - \Delta^2\widehat{\bar{S}\bar{S}_{ij}}\right). \tag{3}$$

In addition, the denominator of (2) can vanish only if each of the five independent components of M_{ij} vanishes simultaneously. However, the above local unaveraged version (2) still encounters numerical instabilities. The formulation for C actually used in LES is

$$C = \frac{\langle L_{kl}M_{kl}\rangle}{2\langle M_{ij}M_{ij}\rangle}, \tag{4}$$

where the numerator and denominator are now averaged over a plane parallel to the wall. They are assumed to be functions of the distance y normal to the wall and the time t only.

2.2. *Modified dynamic subgrid-scale model*

Several problems arising from the application of the Germano–Lilly dynamic SGS model to low-Reynolds-number LES have been recognized recently,[10,11] such as a greater sensitivity to the choice of the ratio of the test filter width to the grid filter width and a higher magnitude of C than the commonly used value for channel flows. A modification of the Germano–Lilly model must be introduced to obtain better performance of the dynamic SGS model in the low-Reynolds-number situation. Before doing so, some issues regarding the use of the dynamic model in LES are addressed:

(a) the appropriate form of averaging to remove the numerical instability
(b) the choice of the test filter width
(c) the ability to account for energy backscatter.

First of all, when homogeneous directions exist in the turbulent flow concerned, an average over the homogeneous direction or plane is fairly effective in dealing with the numerical instabilities, as demonstrated by other researchers[1,12] and by the present study. In other flows without any homogeneous direction, other averaging choices such as time averaging may be more appropriate. More severe problems, such as the instability attributed to negative total viscosity (the sum of the molecular and the eddy viscosity) sustained over many time steps at some position, may be remedied by artificially setting the total viscosity to zero at those locations, though at the price of losing desirable features of the dynamic SGS model (4) to be discussed below. The use of this remedy should be reduced to a minimum for a better dynamic SGS model.

The second issue arises from the fact that if the ratio of the test filter width to the grid filter width is too small, the dynamic model uses little information on scales of motion between the test scale and the grid scale. In that case the model is not considered to be reliable, since the elements of $L_{ij} = \widehat{\bar{u}_i\bar{u}_j} - \hat{\bar{u}}_i\hat{\bar{u}}_j$ involved in the dynamic model are closely associated with these scales of motion. On the other hand, if it is too large, important local information is averaged away, since some type of average has to be used to compute the flow variables on the test grid from the grid flow field. Thus the requirement for a better dynamic model should be that the important local information be kept as much as possible. Some researchers have even tried to create a local dynamic model.[13]

Thirdly, in the authors' opinion the ability of the SGS dynamic model to account for backscatter may be partially reduced by artificially setting the total viscosity to zero wherever this value becomes negative, thereby losing one conceptual advantage of the dynamic formulation, namely the ability to add randomness to the explicit scales to account for the upgrid energy transfer. A better dynamic model should not predict a large negative value of eddy viscosity over a prolonged period, to avoid the need to artificially set the total viscosity to zero.

Taking the above arguments into consideration, a refinement to the Germano–Lilly model is introduced to give the model coefficient the form

$$C(y, t) = \frac{1}{2}\left\langle \frac{L_{kl}M_{kl}}{M_{ij}M_{ij}} \right\rangle. \tag{5}$$

The important feature of this modified model is that the model coefficient is truly computed on a local basis. Then the dynamically calculated model coefficient, which is time-dependent and varies in space, is averaged over the homogeneous plane parallel to the wall. By construct, in the Germano–Lilly dynamic SGS model (4) the numerator and denominator are averaged over the homogeneous plane separately. That means they have been assumed to be functions of y and t only, before the model coefficient is computed. Thus the resulting model coefficient from the averaged numerator divided by the averaged denominator is longer on a local basis.

The advantage of the modification is that more local information can be kept than in the Germano–Lilly dynamic SGS model. The local information is very important for the quality of the dynamic model. In addition, the physical meaning of the averaging procedure used in (5) is sounder and more straightforward than that in (4). We suggest that this modified dynamic SGS model may be superior to the Germano–Lilly model. Its performance is tested to evaluate the effectiveness of the modification in the following section.

3. APPLICATION TO LOW-REYNOLDS-NUMBER CHANNEL FLOW

3.1. Numerical simulations

The evaluation of the modified dynamic model was carried out in a channel flow. The simulated flow is driven by a constant pressure gradient between parallel infinite walls. The friction Reynolds number Re_τ^+, based on the mean friction velocity u_τ and half-channel width, is 205. In Figure 1 the flow geometry and naming conventions are shown. The box dimensions are $4\pi\delta \times 2\pi\delta \times 2\delta$ in the streamwise (x), spanwise (z) and cross-stream (y) directions respectively, where 2δ is the distance between the walls. Since the fully developed turbulent channel flow is homogeneous in the x- and the z-direction, periodic boundary conditions are used in both these directions. In the direction normal to walls a natural, no-slip boundary condition is applied, since the boundary layer is to be resolved explicitly well inside the linear sublayer.

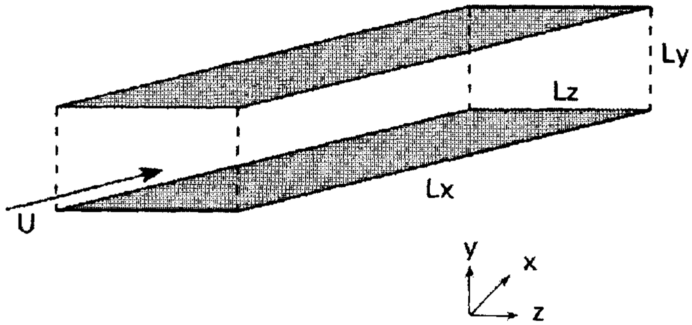

Figure 1. Flow geometry

A finite volume LES code, originally written by Gavrilakis *et al.*[14] and modified to suit the present study, was used here. The code is capable of simulating three-dimensional time-dependent turbulent channel flows having two statistically homogeneous dimensions. The third dimension can involve strong shear. In this code the incompressible momentum equations are discretized by central differences, the Adams–Bashforth time integration scheme is applied to the non-linear, viscous and pressure terms and the pressure at the advanced time is solved from a Poisson equation through Fourier transformation with respect to the streamwise and spanwise homogeneous directions.

Four simulations were performed. The characteristics of these simulations are summarized in Table I. Δ and $\hat{\Delta}$ in Table I are the grid and test filter widths respectively. The filtered velocity fields at these two levels are obtained via discretization used as an implicit filter, rather than an explicit filter, like the Gaussian or the sharp Fourier cut-off filter. Following Deardorff,[15] in the present study they are defined as

$$\Delta(\Delta_x\Delta_y\Delta_z)^{1/3}, \qquad \hat{\Delta} = (\hat{\Delta}_x\hat{\Delta}_y\hat{\Delta}_z)^{1/3}. \tag{6}$$

For comparison, runs A2 and A3 use the Germano–Lilly dynamic SGS model. By contrast, runs B2 and B3 use the modified dynamic SGS model. To investigate the sensitivity of the simulation results to the choice of the ratio of the test filter width to the grid filter width, in A2 and B2 we have $\hat{\Delta}_x = 2\Delta_x$, $\hat{\Delta}_y = \delta_y$,

Table I. Characteristics of simulations and models

symbol	A2	A3	B2	B3
$L_x \times L_y \times L_z$	$4\pi\delta \times 2\delta \times 2\pi\delta$			
$N_x \times N_y \times N_z$	$96 \times 64 \times 80$			
Δ_x^+	$26\nu/u_\tau$			
$(\Delta_y^+)_{\min}$	$0{\cdot}635\nu/u_\tau$			
$(\Delta_y^+)_{\max}$	$15{\cdot}22\nu/u_\tau$			
Δ_z^+	$15{\cdot}5\nu/u_\tau$			
Model type	$C(y,t) = \dfrac{\langle L_{kl}M_{kl}\rangle}{2\langle M_{ij}M_{ij}\rangle}$		$C(y,t) = \dfrac{1}{2}\left\langle \dfrac{L_{kl}M_{kl}}{M_{ij}M_{ij}} \right\rangle$	
$\hat{\Delta}/\Delta$	$2^{2/3}$	2	$2^{2/3}$	2

$\hat{\Delta}_z = 2\Delta_z$ and therefore $\hat{\Delta}/\Delta = 2^{2/3}$, whereas in A3 and B3 we have $\hat{\Delta}_x = 2\Delta_x$, $\hat{\Delta}_y = 2\Delta_y$, $\hat{\Delta}_z = 2\Delta_z$ and therefore $\hat{\Delta}/\Delta = 2$. The base model used by both the Germano–Lilly and the modified dynamic model is the Smagorinsky model. These four runs were all started from the instantaneous state of a large-eddy simulation ($96 \times 64 \times 80$) at $t = 101{\cdot}5\delta/u_\tau$.[11] The time step was $0{\cdot}0003\delta/u_\tau$ for each simulation. Run A2 required 11·313 s per time step on a Cray Y-MP8, run B2 1·5079 s, run A3 2·3217 s and run B3 2·3271 s. It can be seen that A2 is most costly in terms of computer time among the four runs. The reason for this will be given later.

3.2. Results and discussion

The computed model coefficients C for runs A2, A3, B2 and B3 are presented in Table II. Note that the model coefficient obtained from run A2 at some locations became negative or very large. Figure 2 illustrates the turbulence length scales $C(\Delta_x\Delta_y\Delta_z)^{1/3}$ computed from the results of runs A2 (open triangles), A3 (full triangles), B2 (open circles) and B3 (full circles) in comparison with the length scale

$$l_{\mathrm{MK}} = C_s[1 - \exp(-y^+/A^+)](\Delta_x\Delta_y\Delta_z)^{1/3} \tag{7}$$

(broken curve) proposed by Moin and Kim[16] (referred to as MK), the length scale

$$l_{\mathrm{PFM}} = C_s[1 - \exp(-y^{+3}/A^{+3})]^{1/2}(\Delta_x\Delta_y\Delta_z)^{1/3} \tag{8}$$

(full curve) suggested by Piomelli *et al.*[17] (referred to as PFM) and the $y^{+3/2}$ curve (dotted line), which is the correct asymptotic behaviour of the turbulence length scale near the wall. (The negative value in run A2 has been removed to plot the logarithmic profile.) Note that in (7) and (8), $C_s = 0{\cdot}1$ is the optimum value for channel flows.

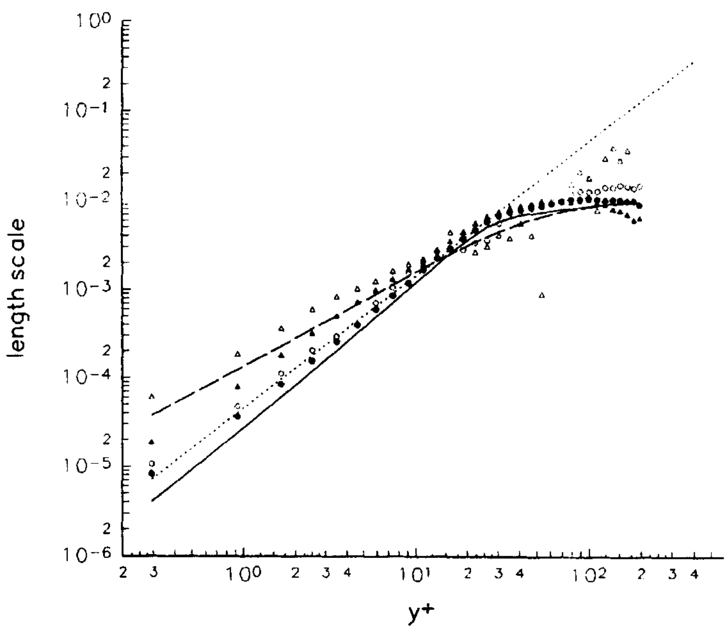

Figure 2. Subgrid length scale C versus y^+: full circles, B3; open circles, B2; full triangles, A3; open triangles, A2; full curve, $0{\cdot}1[1 - \exp(-y^{+3}/A^{+3})]^{1/2}(\Delta_x\Delta_y\Delta_z)^{1/3}$; broken curve, $0{\cdot}1[1 - \exp(-y^+/A^+)](\Delta_x\Delta_y\Delta_z)^{1/3}$; dotted line, $y^{+3/2}$ curve

Table II. Dynamically computed model coefficients C

y^+	A2	A3	B2	B3
0·2946	0·3894E-05	0·3698E-06	0·2052E-06	0·8525E-07
0·9293	0·3207E-04	0·5903E-05	0·3585E-05	0·1473E-05
1·6620	0·1151E-03	0·2763E-04	0·1746E-04	0·7203E-05
2·5070	0·2787E-03	0·7946E-04	0·5409E-04	0·2224E-04
3·4820	0·4938E-03	0·1759E-03	0·1381E-03	0·5500E-04
4·6050	0·6633E-03	0·3334E-03	0·3596E-03	0·1208E-03
5·8990	0·9161E-03	0·5740E-03	0·4192E-03	0·2493E-03
7·3860	0·1440E-02	0·9291E-03	0·9276E-03	0·4906E-03
9·9060	0·1817E-02	0·1438E-02	0·2304E-02	0·9151E-03
11·060	0·1721E-02	0·2169E-02	0·2491E-02	0·1582E-02
13·310	0·3230E-02	0·3152E-02	0·2459E-02	0·2551E-02
15·880	0·7171E-02	0·4552E-02	0·5463E-02	0·3710E-02
18·820	0·6817E-02	0·6675E-02	0·8316E-02	0·5139E-02
22·170	0·2114E-02	0·9625E-02	0·1001E-01	0·7018E-02
25·980	0·2647E-02	0·1272E-02	0·2088E-01	0·8891E-02
30·300	0·4376E-02	0·1523E-02	0·2324E-01	0·1066E-01
35·180	0·3426E-02	0·1697E-02	0·2496E-01	0·1197E-01
40·680	0·6948E-02	0·1742E-01	0·2310E-01	0·1287E-01
46·860	0·3370E-02	0·1792E-01	0·2671E-01	0·1379E-01
53·760	0·1545E-03	0·1977E-01	0·2552E-01	0·1481E-01
61·430	−0·3090E-02	0·1932E-01	0·2380E-01	0·1573E-01
69·920	0·1434E-01	0·1685E-01	0·2679E-01	0·1633E-01
79·250	0·3699E-01	0·1634E-01	0·2913E-01	0·1621E-01
89·440	0·6638E-01	0·1455E-01	0·2940E-01	0·1686E-01
100·50	0·4586E-01	0·1478E-01	0·2986E-01	0·1672E-01
112·40	0·8154E-02	0·1514E-01	0·3260E-01	0·1479E-01
125·10	0·1153	0·1272E-01	0·3279E-01	0·1452E-01
138·50	0·1847	0·1021E-01	0·2179E-01	0·1377E-01
152·60	0·9771E-01	0·8141E-02	0·2310E-01	0·1269E-01
167·10	0·1568	0·7339E-02	0·2709E-01	0·1104E-01
182·10	−0·4436E-01	0·5931E-02	0·2254E-01	0·9982E-02
197·40	−0·1592	0·4402E-02	0·2925E-01	0·1105E-01

There are several important findings. Firstly, the length scales from runs A2 and A3 using the Germano-Lilly dynamic SGS model (4) are close to the curve of l_{MK}, while the length scales from runs B2 and B3 using the modified dynamic SGS model (5) are close to the curve of l_{PFM}. This is best tuned to ensure a correct behaviour for the length scale of turbulence near the wall among the various existing empirical wall damping functions and therefore is believed to be most capable of giving an accurate length scale profile in the near-wall region.[17] Of great promise is the profile of length scale obtained from run B3, which shows the correct behaviour of the subgrid eddy viscosity in the region of a plane wall to an accuracy that exceeds this best-tuned wall damping function (PFM), and almost collapses with the curve of $y^{+3/2}$, the expected behaviour of the length scale near the wall.

Secondly, run A2 using the Germano–Lilly dynamic SGS model (4) with $\hat{\Delta}/\Delta = 2^{2/3}$ encountered numerical instability, whereas run A3 using the same SGS model but with $\hat{\Delta}/\Delta = 2$ is stable. This indicates that the Germano–Lilly dynamic SGS model (4) shows a greater sensitivity to the value of $\hat{\Delta}/\Delta$ for low-Reynolds-number channel flow. A greater sensitivity to the ratio $\hat{\Delta}/\Delta$ has also been encountered by Cabot and Moin[10] in their simulations of low-Reynolds-number channel flows using a dynamic SGS model in the form (4). In contrast, as can be seen clearly from the length scale profiles obtained from B2 and B3 using the modified dynamic SGS model (5), the changes in the value of $\hat{\Delta}/\Delta$ did not affect the results significantly.

Thirdly, in run A2 the numerical instability encountered was remedied by artificially setting the total viscosity $(\nu + \nu_t)$ to zero at those locations where negative total viscosity occurred, following Akselvoll and Moin.[3] As already presented earlier in this section, run A2 was most costly, approximately 7·5 times more expensive than B2 and even 4·9 times more expensive than A3 and B3. (This was certainly due to poor vectorization of the part of the DSGM coding checking the locations where the total viscosity was negative and setting it to zero.) Runs A3 and B3, with $\hat{\Delta}/\Delta = 2$, involve more neighbouring grid points and therefore more computation than A2 and B2 when the velocity field is filtered by the test filter whose width is larger in A3 and B3 than in A2 and B2.

Finally, the low-Reynolds-number LES results for channel flow performed in this study (Table II) show that the largest magnitude of the eddy viscosity computed dynamically for run A3 using the Germano–Lilly model is 41% higher than 0·1, the value commonly used for channel flows. However, in run B3 using the modified dynamic model, the largest magnitude of the eddy viscosity is larger than 0·1 by 27% and the region where the larger eddy viscosities occur is smaller. This indicates that the modification has brought about an alleviation of the deviation of the computed value from the commonly used one, although the magnitude remains high, since we continued to use the Smagorinsky model as the base model in deriving the modified dynamic SGS model.

4. CONCLUSIONS

The simulations performed in this study indicate that the advantages of the modified dynamic SGS model (5), which keeps more of the local information believed to be important for the reliability and accuracy of the dynamic SGS model, can bring about an improvement in dynamic SGS modelling. The success of the modified dynamic SGS model in low-Reynolds-number LES for channel flow is threefold. Firstly, the resulting length scale from LES using the modified model has shown an almost perfect behaviour in the region of a plane wall without any need to adjust the model coefficient or to invoke *ad hoc* wall damping functions. In contrast, the performance of the Germano–Lilly dynamic SGS model is marginal. Secondly, for the modified model the choice of $\hat{\Delta}/\Delta$ did not affect the predicted length scale significantly, whereas for the Germano–Lilly model the choice of $\hat{\Delta}/\Delta$ has a considerable impact on the predicted length scale. Finally, the modified model illustrates an improved numerical stability compared with the Germano–Lilly model.

The results of LES using the modified dynamic SGS model show great promise. However, there still exists a problem related to its application to low-Reynolds-number flow. Despite being much lower than that resulting from the Germano–Lilly dynamic SGS model, the magnitude of the eddy viscosity computed dynamically by the modified dynamic model remains high. This deficiency may arise from the assumption used by Germano to derive the dynamic model, namely that the same formulation (the standard Smagorinsky model) can be used to parametrize SGS stresses at both grid and test levels and therefore the grid and test scale cuts are limited to an inertial range. Unfortunately, Germano's assumption is inappropriate for low-Reynolds-number LES. In low-Reynolds-number LES or in LES with high enough resolution the grid and test scale cut-offs fall in the dissipation range, in which the energy spectrum is much steeper than that of the inertial subrange. Under this circumstance the Smagorinsky formulation overestimates the energy transfer across the cut, which in turn results in overprediction of the magnitude of the eddy viscosity.

To overcome this deficiency, the base model used to derive the formulation of the dynamic SGS model could be one extended to the low-Reynolds-number situation. Such a model, called a dissipation range SGS model, has been proposed[18] and has been tested in low-Reynolds-number LES for channel flow, showing better agreement with DNS and experimental results.[11] A new dynamic SGS formulation based on this dissipation range SGS model has been suggested.[11] More tests need to be done on such a dynamic SGS model.

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